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Addendum to Reconciling Grand Unification with Strings by Anisotropic Compactifications

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In a recent paper [1], by working in the orbifold GUT limit of the Heterotic string, we showed how one could accommodate gauge coupling unification in the "mini-landscape" models of References [2–6]. Furthermore, it was shown how one of the solutions was consistent with the decoupling of other exotics and $F = 0$. In this short addendum, we show that this solution is also consistent with $D = 0$.

Let us first describe the steps one must take to show that there is a solution to the equations $F = D = 0$. For simplicity, and without loss of generality, we will consider a $\mathrm{U}(1)_A \times \mathrm{U}(1)_B$ gauge theory. We will further consider N fields Φ_i charged under both U(1)s, where each Φ_i has charge q_i^A under the first U(1), and charge q_i^B under the second U(1). If we turn the superpotential off, unbroken supersymmetry (SUSY) requires

$$D_A \equiv \sum_{i=1}^{N} q_i^A \, |\Phi_i|^2 = 0 \tag{1}$$

$$D_B \equiv \sum_{i=1}^{N} q_i^B \, |\Phi_i|^2 = 0. \tag{2}$$

It is well known that the moduli space of $D = 0$ is spanned by a basis of holomorphic, gauge invariant monomials (HIMs) [7]. Quite generally, the dimension of the moduli space $\mathcal{M}$ of some gauge group $\mathcal{G}$ is given by the number of fields charged under $\mathcal{G}$ minus the number of constraints coming from $V_D = 0$:

$$\dim \mathcal{M} = N - \dim \mathcal{G}. \tag{3}$$

The HIMs can be represented as vectors $\vec{x}^\alpha$ in the $U(1)_A \times U(1)_B$ charge space. That is, if we define the charge matrix $\mathbb{Q}$ as

$$\mathbb{Q} \equiv \begin{pmatrix} q_1^A & q_2^A & \cdots & q_N^A \\ q_1^B & q_2^B & \cdots & q_N^B \end{pmatrix}, \tag{4}$$

then the set of HIMs are defined by the solutions to

$$\mathbb{Q} \cdot \vec{x}^\alpha = 0. \tag{5}$$

The requirement that the monomials be holomorphic is a non-trivial constraint—effectively, this means that the entries in $\vec{x}^\alpha$ be positive semi-definite integers.[1] The set of $\vec{x}^\alpha$s are linearly independent N dimensional vectors spanning a space with $\dim \mathcal{M} = N - \dim \mathcal{G}$. Then the set of monomials spanning the moduli space $\mathcal{H}$ is given by

$$\mathcal{H} = \{ M_\alpha = \Phi_1^{x_1^\alpha} \Phi_2^{x_2^\alpha} \cdots \Phi_N^{x_N^\alpha} \}. \tag{6}$$

We can guarantee solutions to $V_D = 0$ if we demand that

$$|\Phi_i|^2 = \Phi_i \frac{\partial}{\partial \Phi_i} \sum_\alpha^{\dim \mathcal{M}} a_\alpha M_\alpha. \tag{7}$$

In general, one must choose the constants a_α such that all of the phases on the right hand side of Equation (7) cancel. A substitution into Equations (1) and (2) shows that we do indeed satisfy $D = 0$.

Next, consider the case where we turn on a superpotential, $\mathcal{W}$. The superpotential is an arbitrary function of the holomorphic, gauge invariant monomials given by $\mathcal{W} = \mathcal{W}(M_\alpha)$. The requirement that $F = 0$ can be stated as follows:

$$\Phi_i \frac{\partial}{\partial \Phi_i} \mathcal{W}(M_\alpha) = 0. \tag{8}$$

This only tells us something that we already knew—the superpotential only constrains combinations of the holomorphic, gauge invariant monomials M_α, not the fields themselves.[2] We can solve these constraints explicitly for the M_α, and then express $|\Phi_i|^2$ in terms of a linear combination of the M_α (as before), with arbitrary a_α. Thus we see that it is *always* possible to satisfy $D = 0$ when given a solution to $F = 0$.[3]

Finally we consider the (relevant) case where the "A" in $U(1)_A \times U(1)_B$ stands for "anomalous". In this case, Equation (1) is modified slightly. We must now cancel a Fayet-Iliopolous (FI) term if we wish to keep SUSY unbroken:

$$D_A = \sum_i |\Phi_i|^2 + |\xi| = 0. \tag{9}$$

[1] Here we will note that it is entirely possible that the null space of the charge matrix Q is empty—that is, it could very well be that there exists no holomorphic, gauge invariant monomials. This corresponds to a situation where SUSY is broken spontaneously *everywhere* in moduli space by D terms, except possibly at the origin where one would expect an enhanced gauge symmetry.

[2] See Chapter VIII in Reference [8], for example.

[3] A different argument was made by Luty and Taylor [9].

In order to ensure that there exists a direction in moduli space along which this constraint can be satisfied, we seek at least one HIM which has a net negative charge under $U(1)_A$. This will ensure that we can cancel the (negative) FI term.

We now turn to the issue which we would like to address, namely proving $D = 0$ for the solution presented in Section 4 of Reference [1], wherein it was shown that $F = 0$ for one of the models, but the issue of $D = 0$ was neglected. We consider Model 1A in Reference [6], where it was shown that solutions to $F = D = 0$ existed for arbitrary (string scale) vevs for some subset of the non-Abelian singlet fields—see Equation (5.3) of [6]. However for gauge coupling unification we require two fields, called s_1 and s_{25}, to have intermediate scale (M_{EX}) vevs, while several other fields are required to have vevs of order the string scale [1]. There are also several other non-Abelian singlet fields which we require to get zero vevs. (The complete list of fields, along with their charges are listed in Appendix E of Reference [6].)

Using the arguments above, we note that the proof of $D = 0$ for our solution is straightforward. One only need check that there are enough HIMs, including all fields, to saturate the dimension of the moduli space, and that there exists at least one holomorphic monomial, excluding the fields s_1 and s_{25}, which has a negative charge under the $U(1)_A$. We have verified that this is the case.

In closing, we will note that simply taking s_1 and s_{25} out of the charge matrix $\mathbb{Q}$ produces a null result for $\mathbb{Q} \cdot \vec{x}^{\alpha} = 0$, meaning that there are no vectors $\vec{x}^{\alpha}$ which satisfy the above equation if we set the vevs of $s_1 = s_{25} = 0$. We point out that the solution to $F = 0$ does require one engineer a cancellation on the order $M_{\text{EX}}/M_{\text{S}} \sim 10^{-8}$ among the other vevs. We should expect, then, that a tuning in the coefficients a_α is required to this order as well. While this is aesthetically unappealing, it is nonetheless possible, as the relationship in Equation (7) only constrains the phases of the a_αs and not their magnitudes.

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References

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